Fill in the following identities. **SCORE: _____ / 14 PTS**

[a] POWER REDUCING IDENTITY:

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

[b] HALF ANGLE IDENTITY:

$\cos\frac{1}{2}x = \pm\sqrt{\frac{1+\cos x}{2}}$

[c] PYTHAGOREAN IDENTITY:

$\tan^2 x = \sec^2 x - 1$

[d] NEGATIVE ANGLE IDENTITY:

$\sec(-x) = \sec x$

[e] DIFFERENCE OF ANGLES IDENTITY:

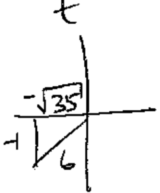
$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

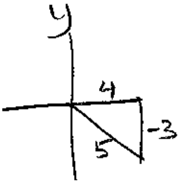
[f] SUM OF ANGLES IDENTITY:

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

[g] DOUBLE ANGLE IDENTITY: $\cos 2x = \cos^2 x - \sin^2 x$ or $2\cos^2 x - 1$ or $1 - 2\sin^2 x$

WRITE ALL 3 VERSIONS


t
$-\sqrt{35}$
-1
6


y
4
-3
5

$\tan\frac{1}{2}t$

$$= \frac{1-\cos t}{\sin t}$$

$$= \frac{1-\left(-\frac{\sqrt{35}}{6}\right)}{-\frac{1}{6}} \cdot \frac{-6}{-6}$$

$$= -6-\sqrt{35}$$

$$\sin(\underbrace{\arctan(-\tfrac{3}{4})}_{y} - t)$$

$$= \sin y \cos t - \cos y \sin t$$

$$= -\frac{3}{5} \cdot -\frac{\sqrt{35}}{6} - \frac{4}{5} \cdot \frac{-1}{6}$$

$$= \frac{3\sqrt{35}+4}{30}$$

tan 2*t*

$$= \frac{2\tan t}{1-\tan^2 t}$$

$$= \frac{2\left(\frac{1}{\sqrt{35}}\right)}{1-\left(\frac{1}{\sqrt{35}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{35}}}{\frac{34}{35}}$$

$$= \frac{2}{\sqrt{35}} \cdot \frac{35}{34_{17}}$$

$$= \frac{\sqrt{35}}{17}$$

Solve the equation $9-7\cos 5x=5(2-\cos 5x)$. **SCORE: _____ / 14 PTS**

$$9-7\cos 5x = 10-5\cos 5x$$

$$-2\cos 5x = 1$$

$$\cos 5x = -\frac{1}{2}$$

$$5x = \frac{2\pi}{3}+2n\pi \text{ or } \frac{4\pi}{3}+2n\pi,\ n\in\mathbb{Z}$$

$$x = \frac{2\pi}{15}+\frac{2n\pi}{5} \text{ or } \frac{4\pi}{15}+\frac{2n\pi}{15},\ n\in\mathbb{Z}$$

Prove the identity $\cot^2 x + \sec^2 x = (\csc x - \tan x)^2 + 2\sec x$. **SCORE: _____ / 14 PTS**

$$\hookrightarrow = \csc^2 x - 2\csc x \tan x + \tan^2 x + 2\sec x$$

$$= \cot^2 x + 1 - 2\frac{1}{\sin x}\frac{\sin x}{\cos x} + \sec^2 x - 1 + 2\sec x$$

$$= \cot^2 x + \sec^2 x$$

QED

Rewrite $\sin^4 x \cos^2 x$ using only the first powers of cosine (and constants and the 4 basic arithmetic operations). **SCORE: _____ / 14 PTS**
Simplify your final answer, which must **NOT** be in factored form, and must **NOT** involve any other trigonometric functions.

$$= \left[\tfrac{1}{2}(1-\cos 2x)\right]^2 \cdot \tfrac{1}{2}(1+\cos 2x)$$

$$= \tfrac{1}{8}(1-\cos 2x)(1-\cos 2x)(1+\cos 2x)$$

$$= \tfrac{1}{8}(1-\cos 2x)(1-\cos^2 2x)$$

$$= \tfrac{1}{8}(1-\cos 2x)(1-\tfrac{1}{2}(1+\cos 4x))$$

$$= \tfrac{1}{8}(1-\cos 2x)(\tfrac{1}{2}-\tfrac{1}{2}\cos 4x)$$

$$= \tfrac{1}{16}(1-\cos 2x)(1-\cos 4x)$$

$$= \tfrac{1}{16} - \tfrac{1}{16}\cos 2x - \tfrac{1}{16}\cos 4x + \tfrac{1}{16}\cos 2x \cos 4x$$

Solve the equation $3(4+\cos 2x)=5(3+\cos x)$ algebraically. **SCORE: _____ / 14 PTS**

Round your answers to 4 decimal places.

$12+3\cos 2x = 15+5\cos x$

$12+3(2\cos^2 x-1)=15+5\cos x$

$6\cos^2 x+9=15+5\cos x$

$6\cos^2 x-5\cos x-6=0$

$(3\cos x+2)(2\cos x-3)=0$

$\cos x = -\frac{2}{3}$ OR ~~$\frac{3}{2}$~~

$x_{REF}=\cos^{-1}\frac{2}{3}\approx 0.8411$

$x\in Q_2$ OR Q_3

$x=\pi-0.8411+2n\pi$ OR $\pi+0.8411+2n\pi$, $n\in\mathbb{Z}$

$=2.3005+2n\pi$ OR $3.9827+2n\pi$, $n\in\mathbb{Z}$